

Tiebreakers

AMSA-MAMS Pi Day Mathematics Tournament

March 9, 2019

1. Scrooge McDuck is being held hostage by an angry mathematician. The mathematician writes five distinct numbers a, b, c, d, e on the board. The mathematician knows that by Szekeres's theorem, there are three numbers that, in order, are either all increasing or decreasing (e.g., $a < b < d$ or $b > c > e$, but not $e < a < c$). Scrooge is blindfolded so that he can't see the numbers, but if he can tell the mathematician which numbers satisfy these properties. Luckily, he can bribe the guard *once* for $\$X$. Then, the guard will answer X questions about the order of the numbers (for example, $a > b$ or $b < e$), one by one. What's the smallest amount of money Scrooge must pay in order to guarantee his escape?

2. Scrooge McDuck is in trouble with a mathematician again. The mathematician writes five numbers $a < b < c < d < e$ on the board. Scrooge is blindfolded, but he will be freed if he can tell the mathematician whether or not π is written on the board. Luckily, he can bribe the guard *once* for $\$X$. Then, the guard will answer X questions strictly comparing π to a number on the board (for example, $\pi < a$ or $\pi > d$, but not $\pi = b$ or $\pi \leq c$), one by one. What's the smallest amount of money Scrooge must pay in order to guarantee his escape?

3. Scrooge McDuck keeps getting in trouble. The mathematician writes six distinct numbers such that they first increase, then decrease (for example, $a > b > c > d > e > f$ or $a < b < c < d > e > f$). Scrooge is blindfolded, but he will be freed if he can tell the mathematician which of a, b, c, d, e, f is the largest. Luckily, he can bribe the guard *once* for $\$X$. Then, the guard will answer X questions about the order of the numbers (for example, $a > b$ or $b < e$), one by one. What's the smallest amount of money Scrooge must pay in order to guarantee his escape?