β Beta Round Solutions

AMSA-MAMS Pi Day Mathematics Tournament

March 10, 2018

1. How many square yards of carpet are required to cover a rectangular floor that is 24 feet long and 18 feet wide?

   Answer: $\boxed{48}$ 24 feet = 8 yards and 16 feet= 6 yards. $8 \cdot 6 = 48$ square yards

2. What is $0.\overline{2} + 0.0\overline{2} + 0.00\overline{2}$? Express your answer as a common fraction.

   Answer: $\boxed{\frac{37}{150}}$ $0.\overline{2} + 0.0\overline{2} + 0.00\overline{2} = \frac{2}{9} + \frac{2}{90} + \frac{2}{900} = \frac{37}{150}$

3. After shooting 30 times, a basketball player makes 40% of her shots. What is the minimum number of shots she needs to take to raise her shooting accuracy to 50%?

   Answer: $\boxed{6}$ $0.4 \cdot 30 = 12$ wins

$$\frac{12+x}{30+x} = \frac{1}{2}$$

$$24 + 2x = 30 + x$$

$$x = 6$$

4. A positive integer n is called *nice* if the number is composed of only the digits 3 and 6. Find the smallest *nice* number divisible by 64.

   Answer: $\boxed{6336}$ Note that the number must be greater than a one digit number. So, n cannot be 3 or 6. By going through all the possible combinations, one may find that the smallest such number is 6336.

5. If it takes 10 workers 50 minutes to make 36 toys, how long (in minutes) would it take 15 workers to make 27 toys?

   Answer: $\boxed{25}$ 10 workers 50 minutes 36 toys $\implies$ 1 worker 50 minute $\frac{36}{10}$ toys $\implies$ 1 worker $\frac{50}{\frac{36}{10}}$ minutes 1 toy $\implies$ 1 worker $\frac{500}{36}$ minutes 1 toy $\implies$ 15 workers 25 minutes 27 toys

6. A case contains candy. If the candies are equally divided among 7 people, then 4 candies are left over. If the candies are equally divided among 11 people, then 7 candies are left over. Assuming that there are the least amount of candies satisfying both conditions, how many candies are left when divided among 17 people

   Answer: $\boxed{1}$ The smallest number of candies possible is 18. When divided among 17 people, only 1 is left

7. What is the smallest natural number k such that there does not exist a natural number n such that $n!$ (or n factorial) has k ending zeroes?

   Answer: $\boxed{5}$ Consider the positive multiples of 5: $5, 10, 15, 20, 25, \ldots$ Because the product of the first k positive multiples of 5 never has 5^5 as the highest power of 5 dividing it, the smallest such natural number must be 5.

8. Aditya is particularly fond of Tom Brady. Because of this, he finds the value of 12!+11!+...+2!+1!. He then finds the sum of the digits of that value. He then finds the sum of the digits of that value. He continues this process until he is left with a single digit number. What number is he left with?

   Answer: $\boxed{9}$ This question is essentially asking what the original sum mod 9 is. For $n > 5$, $n!$ is divisible by 9. 5!+4!+3!+2!+1!=0 mod 9, so the single digit number would be 9.

9. If in the expansion of $(1+x)^m(1-x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, calculate the value of $m+n$.

   Answer: $\boxed{15}$ $(1+x)^m = 1 + mx + \frac{m(m-1)}{2}x^2 + ...$, $(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^+....$ Multiplying, we get $m - n = 3$ and $\frac{m(m+1)}{2} + \frac{n(n+1)}{2} - mn = -6$. Then, $m^2 + m + n^2 - n - 2mn = -12$ or $(m-n)^2 - (m+n) = -12$. Thus, $m + n = 21$. This gives $m = 12, n = 3$

10. A gumball machine contains 150 green, 200 red, 200 blue, and 50 yellow gumballs. Alex takes some number of gumballs from the machine. Now the probability of reaching into the machine and picking a green gumball is $\frac{1}{7}$, and the probability of picking either a green or red gumball is $\frac{2}{5}$. What is the least number of gumballs that Alex could have removed?

    Answer: $\boxed{215}$ For there to be a $\frac{1}{7}$ chance of picking a green gumball and a $\frac{2}{5}$ chance of picking a red or green gumball, the number of remaining gumballs must be a multiple of 35. So let the number of remaining gumballs be $35x$. Then the number of green gumballs remaining must be $\frac{1}{7} * 35x = 5x$. The number of green and red gumballs remaining must be $\frac{2}{5} * 35x = 14x$, meaning the number of red gumballs is $9x$. This means the least number of gumballs Alex could have removed is $(150 - 5x) + (200 - 9x) = 350 - 14x$, as there could have been blue or yellow gumballs removed as well. We know the total number of gumballs removed is $600 - 35x$, so $350 - 14x$

must be less than or equal to $600 - 35x$. Since we are trying to minimize $600 - 35x$, we are trying to maximize x.

$$350 - 14x < 600 - 35x$$

$$21x < 250$$

Thus, the greatest possible integer value of x is 11, meaning the least number of gumballs Alex could have removed is 600-35*11 = 600-385 = 215.