α Alpha Round Solutions

AMSA-MAMS Pi Day Mathematics Tournament

March 10, 2018

1. There are 5 boys and 5 girls on a math team. How many ways can the coach form a team of 5 if the boys are indistinguishable from the other boys, and the girls are indistinguishable from the other girls?

   Answer: $\boxed{6}$ The only thing that matters is the number of each gender on the team. You can have 0,1,2,3,4, or 5 boys on the team, and any remaining spots will be filled by girls.

2. $243_7 + 4C_{13} = x_{10}$. What is the value of x? Note: a_b denotes a in base b.

   Answer: $\boxed{193}$ $243_7 + 4C_{13} = 129_{10} + 64_{10} = 193_{10}$

3. What is the largest prime factor of $1 + 2 + 3 + \cdots + 160$?

   Answer: $\boxed{23}$ $1 + 2 + 3 + \cdots + 160 = \frac{160 \cdot 161}{2} = 2^4 * 5 * 7 * 23$

4. Let W be the number of wins Kyrie Irving has and L be the number of losses Kyrie Irving has. The least common multiple of W and L is 600, and the greatest common factor of W and L is 20. If $L = 60$, what is W?

   Answer: $\boxed{200}$ $GCF(W, L) \times LCM(W, L) = W \times L$.

5. Srujal is stressing out about the grade he got on his test. In his class of 18 (including him) the lowest grade was a 1 and the highest grade was a 67 (out of 100). The average (arithmetic mean) grade was 13. Assuming everyone got different integer grades, and that Srujal got neither the highest or lowest grade, what is the maximum possible grade Srujal got?

   Answer: $\boxed{31}$ Let Srujal's grade be s, and let everyone else's grades be minimum (so s can be maximum for the average), so the 18 grades are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, s, and 67. Since the average was 13, we can set up the equation

$$\frac{1 + 2 + 3 + \cdots + 16 + s + 67}{18} = 13 \Rightarrow \frac{\frac{16 \cdot 17}{2} + s + 67}{18} = 13$$
$$\Rightarrow 136 + s + 67 = 234 \Rightarrow s = 31$$

6. What is $\sqrt{12+\sqrt{12+\sqrt{12+...}}}$?

   Answer: $\boxed{4}$ If we let $x = \sqrt{12+\sqrt{12+\sqrt{12+...}}}$, then $x^2 = 12 + \sqrt{12+\sqrt{12+\sqrt{12+...}}}$, so $x^2 - x = 12$. So $x = 4$ or $x = -3$. The answer is clearly not negative, so $x = 4$.

7. John predicts the Patriots will have a 93.75% chance of winning at least one Super Bowl in the next 4 years. If the probability that the Patriots win a Super Bowl each year is the same, what is the probability that they win next year? Express your answer as a common fraction.

   Answer: $\boxed{\frac{1}{2}}$ Let the probability that the patriots lose be p. Then, the probability that the patriots lose every superbowl in the next 4 years is p^4. Since this is the compliment of the patriots winning at least one game, $1 - p^4 = 0.9375$

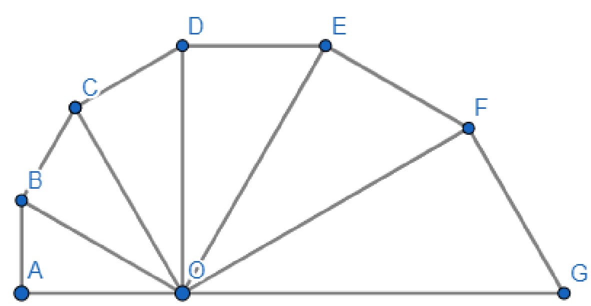
$$p^4 = 0.0625$$

$$p = \frac{1}{2}$$

8. The six right triangles are all similar with right angles at points A B C D E, and F with

   $\Delta AOB \sim \Delta BOC \sim \Delta COD \sim \Delta DOE \sim \Delta EOF \sim \Delta FOG$

   If $AO = \sqrt{3}$ and $AB = 1$, find the length of AG if A, O, and G, are all colinear. Write your answer as a fraction in simplest radical form.



   Answer: $\boxed{\frac{91\sqrt{3}}{27}}$

   $BO = 2$

   $GO = 2 * (\frac{2}{\sqrt{3}})^5 = \frac{64\sqrt{3}}{27}$

   $AG = AO + GO = \sqrt{3} + \frac{64\sqrt{3}}{27} = \frac{91\sqrt{3}}{27}$

9. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and sum of the remaining cards is 1224. If the smaller of the numbers on the removed card is k then calculate $k-20$.

   Answer: $\boxed{5}$ $\frac{n(n+1)}{2} - k - (k+1) = 1224$. Then, $n^2 + n - 4k = 2450$, or $(n+50)(n-49) = 4k$. Let $n = 50$. Then, $k = 25$, so $k - 20 = 5$

10. 4 points are chosen at random from a 5×5 grid of points. What is the probability that the four points are the vertices of a rectangle with sides parallel to the sides of the grid? Express your answer as a common fraction.

    Answer: $\boxed{\dfrac{2}{253}}$ There are $\dbinom{5}{2} = 10$ ways of choosing two rows and $\dbinom{5}{2} = 10$ ways of choosing two columns. Thus, we have $\dfrac{100}{\dbinom{25}{4}} = \dfrac{2}{253}$.