Tiebreaker

AMSA-MAMS Pi Day Mathematics Tournament

March 11, 2017

30 minutes

1. Let x_1, x_2, x_3 be the roots of $x^3 - 6x^2 + 25x + 7$. Compute $2(x_1 + x_2)(x_2 + x_3)(x_3 + x_1)$.

2. Andrew Wiles is trying to climb up 12 stairs. He can take steps of either 1, 2, or 3 stairs at a time. How many different ways can he climb up the steps? One possible way is $3, 3, 3, 3$ Note: The path $2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$ is different than the path $1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2$.

3. Let $\triangle PIE$ be a right triangle such that $PI = 21$ and $IE = 20$ and there is a right angle at I. Let P_1 be the foot of the altitude from I to PE. Construct the altitude from P_1 to IE such that the foot of the altitude is I_1. Continue this process infinitely. If m/n is the sum of the lengths $IP_1, P_1I_1, \ldots$ such that m and n are integers and $\gcd(m, n) = 1$, find the sum of m and n.

4. Danush wants to write his name on a test, but he is unable to spell. How many ways can he misspell his new name so that none of the letters appear in the correct position?