

1. [5] Praneet eats  $\frac{1}{4}$  of a pie in 12 seconds, and Danush eats  $\frac{1}{7}$  of a pie in 7 seconds. Who eats faster?
2. [5] Pie is sold in boxes of 6, 12 and 24 slices. What is the minimum number of boxes needed to buy to get 90 slices of pie?
3. [5] A sphere with diameter  $x$  has a volume of  $\pi$ . Find the volume of a sphere with radius  $x$ .
  
4. [6] Maryam Mirzakhani wants to leave the store. However, in order to do so, she needs to choose the correct door from three choices. Door A reads "I am the correct door." Door B reads "Door C is the correct door." Door C reads "A is the correct door." Exactly one of the doors is telling the truth. Which door should she take?
5. [6] Rishi and Praneet went to a rock concert together. Praneet bought a T-shirt, 2 posters, and a pair of sunglasses at the merch table for \$53. Rishi bought 4 posters and 3 pairs of sunglasses. If sunglasses cost twice as much as posters, and T-shirts cost \$28 more than posters, how much money did Rishi and Praneet spend together?
6. [6] Find the sum of the factors of 60.
  
7. [7] The mean of a set of 6 numbers  $\{18, 50, 62, x - 10, x, 2x\}$  is 38. Find the value of  $x$ .
8. [7] Aditya wants to climb the Leaning Tower of Pisa, which is 200 meters high. Every 45 minutes, he climbs 10 meters, but he needs to rest for 15 minutes, falling 5 meters. How many hours does it take him to climb the tower? Do not round your answer.
9. [7] What is the smallest positive integer that is divisible by the first 10 natural numbers?
  
10. [8] Charan was supposed to write 15 problems for the Pi Day Mathematics Tournament by Sunday. Unfortunately he was only done with 2 problems by Sunday. On Monday, he wanted to make up for his incompetence, so he started writing problems. At the end of the day, he said, "I have written at least a third of the total number of problems everyone, including myself, has written." If there were 87 total problems done by Sunday, including his 2, and no one else wrote problems on Monday, what is the minimum number of problems Charan wrote on Monday?
11. [8] Srinu needs to tie 300 shoelaces. He can tie the shoelaces in 5 hours if he works alone. If Sathwik can tie 150 shoelaces in 5 hours, how many hours will it take them to tie 300 shoelaces if they work together?
12. [8] The decimal expansion of  $1/7$  repeats with period 6. The decimal expansion of  $1/7$  is 0.142857... Find the period of the repeating part of the decimal expansion of  $1/13$ .

13. [9] A coin of radius 1 cm is tossed onto a surface completely tiled with perfect squares with side length 10 cm. What is the probability that the coin lands completely within one of the tiles?
14. [9] Patrick plays video games for 5 minutes on April 1. Every day, he plays 5 more minutes than the day before. For how many minutes does he play video games during the month of April?
15. [9] Let a *not nice* chess board be an 8x8 chessboard with one pair of opposite corners removed. How many ways are there to tile a *not nice* chessboard with 31 standard 2x1 dominoes?
16. [10] Innia needs to write a 5-syllable line for a haiku. Words can be either 1, 2, or 3 syllables. All words with the same number of syllables are identical. In how many ways can she write her line if the order of the words matters?
17. [10] Fermat has 16 feet of fence and wants to make a rectangular pen for his coyotes. He will use the barn as one side of the pen, so the fence only needs to be used for the remaining three sides. What are the dimensions of the pen with the largest area that he can make?
18. [10] Solve  $2^{2^{2^x}} = 16$  where  $x$  is a real number.
19. [11] Cedric Villani wants to watch an hour-long episode of anime while doing his homework. Every hour, he watches one-half of the remaining portion of the episode. How many hours of anime does he watch in 8 hours?
20. [11] Let right triangle  $ABC$  be inside its circumcircle  $P$ . If  $m\angle A = 30^\circ$ ,  $m\angle B = 90^\circ$  and  $AB = 7$ , find the radius of  $P$ .
21. [11] Marie wants to walk home. However, in order to get there, she must navigate a 7 by 4 grid. She begins in the bottom left square and may only move upwards or to the right by one square each step. How many paths can she take to get to the top right square?
22. [12] Andrew Wiles is at the origin of the coordinate plane. Every second, he moves one unit to the right with probability  $1/2$  and to the left with probability  $1/2$ . He wins if he reaches  $(2,0)$  and loses if he reaches  $(-1,0)$ . What is the probability that he wins?
23. [12] Find the largest natural number  $n$  relatively prime to 8 that divides 2016. Note: if two numbers  $a, b$  are relatively prime, then  $\gcd(a, b) = 1$ .
24. [12] What is the smallest integer with 12 positive factors?

25. [13] Terence Tao is opening an ice cream store with 12 flavors to choose from. He is only allowing customers to get 3 or 4 scoops of ice cream. If customers are allowed to repeat flavors, and the order in which the scoops are on the cone doesn't matter, how many possible combinations are there?
26. [13] A perfectly circular pie is cut in a way that one square and one triangle are circumscribed by the crust. Let  $k$  be the largest value of the minimum possible distance between the vertices in degrees. Find the value of  $k$ .
27. [13]  $PPPIEEEE$  is the string formed by using the letter  $P$  3 times, the letter  $I$  1 time, and the letter  $E$  4 times. Compute the number of different arrangements of this string.

28. [14] Let  $ABCD$  be a square of side-length 1. Let  $E$  be the interior point of square  $ABCD$  such that  $DCE$  is an equilateral triangle. Find the measure of angle  $\angle BAE$  in degrees.
29. [14] Find the value of  $x$  if

$$x = \sqrt{2^{\sqrt{2^{\sqrt{2^{\dots}}}}}}$$

30. [14] If  $x + \frac{1}{x} = 5$ , then find the value of  $x^3 + \frac{1}{x^3}$ .

31. [15] In triangle  $AMC$ , let  $\angle M$  be a right angle and  $AM = 20$ ,  $MC = 21$ . Let  $P$  be on side  $AC$  such that  $m\angle CMP = 45^\circ$ . Find  $CP - AP$ .

32. [15] Let

$$x = 6 + \frac{1}{3 + \frac{1}{6 + \frac{1}{3 + \frac{1}{6 + \dots}}}}$$

Find  $(x - 3)^2$ .

33. [15] Let  $P$  be a circle such that point  $P$  is the center of the circle. Let  $E$  be a point outside the circle such that a line tangent to circle  $P$  intersects the circle at point  $A$ . Construct a secant line from  $E$  such that it intersects circle  $P$  at two distinct points  $S$  and  $T$  such that  $S$  is between  $E$  and  $T$ . If the length of  $EA$  is 6 units and the length  $ST$  is 5 units, find the length of  $ES$ .

34. [20] Paul Erdos has two bags of marbles. Bag 1 contains 4 red marbles and 7 blue marbles. Bag 2 contains 5 red marbles and 3 blue marbles. Erdos chooses a bag at random and randomly selects 2 marbles without replacement. What is the probability that he chose Bag 2 if he chooses 1 red marble and 1 blue marble?

35. [20] Marie goes to a bakery. The bar sells pies that costs \$2, \$3, \$4, and \$5. She has a total of \$29. In how many ways can she buy pies, where the order in which the pies are bought does not matter?

36. [20] Estimate the value of the following expression:

$$(\ln(2017))^\pi$$

Let  $x$  be your answer to this question and  $y$  be the correct value. You will receive  $\max(0, \lfloor 20 - 4 \ln |y - x| \rfloor)$  points on this question.