β Beta Round Solutions

AMSA-MAMS Pi Day Mathematics Tournament

March 11, 2017

1. [2] Grigori and Perelman plan to serve dinner to Leonhard and Euler. If Grigori prepares three bowls of noodles per hour and Perelman prepares one bowl of noodles per hour, how many hours will they need to prepare 16 bowls of noodles?

   Answer: $\boxed{4}$ Grigori makes 3 bowls an hour, and Perelman makes 1 bowl an hour, so together, they make 4 bowls an hour. They need to make $8 \times 2 = 16$ bowls, so they need $16 \div 4 = 4$ hours to do it.

2. [3] Aditya, Rishi, and Charan shared a pizza at math team practice on Friday. If Aditya ate $\frac{1}{5}$ of the pizza, Rishi ate $\frac{3}{8}$ of the pizza, and Charan ate $\frac{1}{2}$ of what remained, what fraction of the pizza was left at the end of practice?

   Answer: $\boxed{\frac{17}{80}}$ Anjali and Brian ate $\frac{1}{5} + \frac{3}{8} = \frac{23}{40}$ of the pizza. Therefore, only $\frac{17}{40}$ of the pizza remains. Out of this, Charan ate $\frac{1}{2}$ of the $\frac{17}{40}$ leaving only $\frac{17}{80}$ left

3. [4] Sathwik and Danush are standing next to each other outside in the afternoon, facing towards the sun. Sathwik is 4 feet tall. Danush is 2 feet taller than Sathwik, and Danush's shadow is 6 feet longer than Sathwik's shadow. How long is Sathwik's shadow?

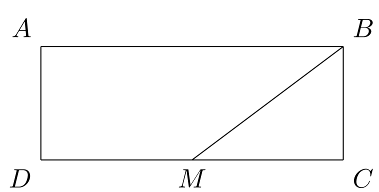
   Answer: $\boxed{12 \text{ feet}}$ $\frac{6}{4} = \frac{3}{2} = \frac{x+6}{x}$, so $x = 12$.

4. [4] Compute the value of $\sqrt{314 + 11 \cdot 2017 - 1}$.

   Answer: $\boxed{150}$ Note that $314 + 11 \cdot 2017 - 1 = 22500 = 150^2$.

5. [5] In rectangle $ABCD$, let M be the midpoint of CD. Let $BM = 5$ and $BC + CM = 7$. Find the area of rectangle $ABCD$.

   Answer: $\boxed{24}$ Refer to the diagram below. Let $BC = x$ and $CM = y$. Then $x^2 + y^2 = 5^2 = 25$ by the Pythagorean Theorem, and so the area of $ABCD$ is $2xy = (x+y)^2 - (x^2+y^2) = 7^2 - 25 = 49 - 25 = 24$.

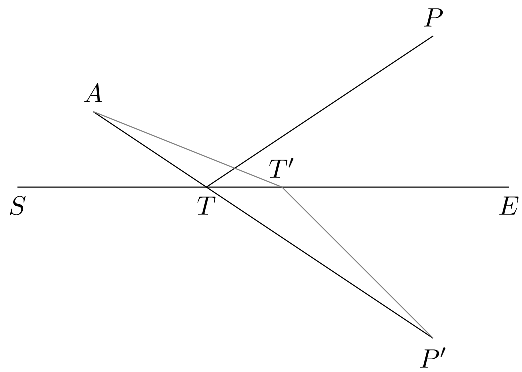


6. [5] Aditya finally got his driver's license and can finally drive to school. His new car has 4 tires and comes with an extra spare tire. He does not want to wear down the tread on his new tires, so he decides to use each tire for an equal amount of distance. If his school is 14 miles away, for how many miles will each tire be used for? Express your answer as a common fraction.

   Answer: $\boxed{\frac{56}{5}}$

7. [6] April and Patrick want to meet along a river at a point O, which is a line segment running from point B to point Q. They both live on the same side of the river. Find the value of $m\angle AOB - m\angle POQ$ in degrees that minimizes the sum of the distances between their homes A and P and the rendezvous location O.

Answer: $\boxed{0}$ Refer to the diagram below. Reflect point P across the line to get P'. Then $P'T = PT$, so $AT + PT = AT + P'T \geq AP'$. By the triangle inequality A, P', and T must be collinear. Then $m\angle P'TE = m\angle PTE = m\angle ATS$, so clearly the answer is 0.



8. [6] The New York Yankees are facing the Boston Red Sox in a 5 game series, where they play all 5 games regardless of the outcomes of the previous games. The Red Sox pitchers for each of the five games are Sale, Porcello, Price, Wright, and Rodgriguez. The Yankees have a 10% chance of beating Sale, a 20% chance of beating Porcello, a 30% chance of beating Price, a 40% chance of beating Wright, and a 50% chance of beating Rodriguez. What is the probability that the Yankees win at least one game? Express your answer as a common fraction.

   Answer: $\boxed{\frac{1061}{1250}}$ The probability the Red Sox win all 5 games is $\frac{9}{10} \times \frac{8}{10} \times \frac{7}{10} \times \frac{6}{10} \times \frac{5}{10} = \frac{189}{1250}$. So, the probability the Yankees win at least one game is $1 - \frac{189}{1250} = \frac{1061}{1250}$.

9. [7] In how many ways can ways a 2×12 chessboard be filled by twelve identical 2×1 dominoes? Rotating and reflecting the dominoes does not change the configuration. For example, one such way is by lying all 12 dominoes vertically.

   Answer: $\boxed{233}$ Lets call F_n The number of ways to fill a 2xn chessboard with n dominoes. $F_1 = 1$ and $F_2 = 2$. A 3x3 chessboard can be filled either by starting with a vertical domino and filling the remaining 2x2 board, or starting with two horizontal dominoes and filling the remaining 1x1 board, so $F_3 = F_2 + F_1$. To generalize, $F_n = F_{n-1} + F_{n-2}$. Working out this Fibonacci sequence gives us $F_{12} = 233$.

10. [8] Anusha is hosting a $\prod$ party for 5 guests. Each of the 6 people bring either a blueberry pie, an apple pie, or a pumpkin pie. If there must be at least one of each type of pie at the party, how many ways are there for the 6 people to bring pies to the party?

    Answer: $\boxed{540}$ Using casework, we see that there are $6\binom{6}{3,2,1} + 3\binom{6}{4,1,1} + \binom{6}{2,2,2} = 6 \cdot 60 + 3 \cdot 30 + 90 = 360 + 90 + 90 = 540$ ways to arrange the food.