

α Alpha Round

AMSA-MAMS Pi Day Mathematics Tournament

March 11, 2017

30 minutes

- [2] Chewing gum is sold in packs of 7, 21 and 28 pieces. Innia buys a total of 91 pieces. What is the smallest number of packs she could have bought?
- [3] Define $x \otimes y = x^2 - y^2 + 2$. Find $(4 \otimes 3) \otimes 3$.
- [4] Find the sum of the mean, median, and mode of 1, 3, 8, 8, 10.
- [4] Suppose that there are 3 distinct digits represented by the symbols A, M, S , all greater than 1. We are given two equations:

$$\begin{array}{rcccc} & A & M & S & A \\ + & M & A & M & S \\ \hline 1 & M & M & 0 & M \end{array}$$

$$A = M + S$$

What is the value of M ?

- [5] The sum of 7 consecutive odd integers is 903. What is the sum of the 6 smallest numbers?
- [5] Choose a_1, a_2, \dots, a_6 without replacement from the set $\{3, 4, 6, 7, 9, 10\}$ and b_1, b_2, \dots, b_6 without replacement from the set $\{1, 2, 5, 8, 11, 12\}$. Find the maximum value of

$$\sum_{i=1}^6 a_i b_i$$

- [6] How many consecutive zeroes does $1212!$ end in? Note: $!$ denotes factorial. For example, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.
- [6] A club has three boys and six girls. In how many ways can a group of five club members be chosen if there must be at least one person of each gender?
- [7] Riemann has n pies that he wants to sell to his friends. If he fills his boxes with 7 pies, there are 6 pies left over. If he fills his boxes with 8 pies, there are 2 pies left over. If he fills his boxes of 9 pies, there are 8 pies left over. Find the smallest possible value of n .
- [8] Let $PIEDAYMT$ be a rectangular prism such that $PIED$ and $YMTA$ are rectangular faces that are parallel to each other so that each vertex of the rectangle $PIED$ is connected with the corresponding vertex on $YMTA$ (P corresponds to Y and so on) by an edge. Let $PD = 3$, $DA = 1$, and $AT = 4$. Also, let the midpoint of AT be K . If PT and IK intersect at point X , compute the length of XK .